Life-Cycle Consumption and Children

Thomas H. Jørgensen,
University of Copenhagen

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Motivation

- **Attanasio, Banks, Meghir and Weber (1999)**: Demographics are important in explaining the consumption profiles (CEX, US)
- **Fernández-Villaverde and Krueger (2007)**: 50% explained by demographics (CEX, US)
- **Browning and Ejrnæs (2009)**: 100% (FES, UK)
Research Question and Contribution

Q: What are the effects of children on non-durable consumption?

It now seems broadly accepted that the effect of children on non-durable consumption is large and important (Attanasio and Weber, 2010)

This study contributes in four ways:

1. Show that the methods previously used in the literature produce flawed results when households face credit constraints.
2. Use existing methods to estimate “rough” bounds.
4. Supply empirical evidence that the effect of children is small.
Outline

1. A Life Cycle Model of Household Consumption
   - Household Composition
2. Euler Equation Estimation
   - Intuition for Inconsistency
   - Constructive Contribution: Bounds
3. Alternative: Structural (M-)Estimator
   - Examples
4. Empirical Applications
   - Danish Register Data
   - US Panel Survey of Income Dynamics (PSID)
5. Concluding Discussion

Consumption and Children, T. Jørgensen.
A Life Cycle Model of Household Consumption

- A standard buffer-stock model of household consumption:

\[
\max_{C_t} \mathbb{E}_t \left[ \sum_{\tau=t}^{T_r-1} \beta^{\tau-t} v(z_t; \theta) u(C_\tau) + \gamma \sum_{s=T_r}^{T} \beta^{s-t} v(z_t; \theta) u(C_s) \right],
\]

- Deterministic retirement at age 60
- Transitory and permanent income shocks
- **A credit constraint** (no net borrowing in baseline)
- CRRA utility: \( u(C_t) = (1 - \rho)^{-1} C_t^{1-\rho} \)
- With multiplicative taste shifter, \( u(C_t) v(z_t; \theta) \), in which \( z_t \) is variables describing household composition and \( \theta \) is their loadings
Household Composition, $v(z_t; \theta)$

- Think of the functional form (applied in most existing studies)

\[ v(z_t; \theta) = \exp(\theta \# \text{ of kids}) \]

(In the empirical application allow for a more flexible functional form)

- **Existing studies**: Estimation of $\theta$ and $(\rho, \beta)$ using reduced form methods based on the *Euler equation*

- **Here**: Structural estimation of $\theta$ and other structural parameters
  - At most three children
  - The arrival of an **infant is stochastic**
  - Children move/do not affect household behavior when turning **21**.
The Constrained Euler Equation

The constrained Euler equation of the model (prior to retirement) is

\[ u'(C_t)v(z_t) - \lambda_t = R\beta \mathbb{E}_t [u'(C_{t+1})v(z_{t+1}) - \lambda_{t+1}] \]

\[ \updownarrow \]

\[ R\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{v(z_{t+1})}{v(z_t)} = \epsilon_{1,t+1} + \epsilon_{2,t+1}, \quad \equiv \epsilon_{t+1} \]  

(1)

where \( \lambda_s \) is the shadow price of resources in period \( s \) and

\[ \mathbb{E}_t[\epsilon_{1,t+1}] = 1, \]
The Constrained Euler Equation

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\[ \uparrow \]

\[ R\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{v(z_{t+1})}{v(z_t)} = \frac{\epsilon_{1,t+1} + \epsilon_{2,t+1}}{\epsilon_{t+1}} \]

(1)

where \( \lambda_s \) is the shadow price of resources in period \( s \) and

\[ \mathbb{E}_t[\epsilon_{1,t+1}] = 1, \]

\[ \epsilon_{2,t+1} = -\frac{\lambda_t - R\beta \mathbb{E}_t[\lambda_{t+1}]}{u'(C_t)v(z_t)}, \]

such that

\[ \mathbb{E}_t[\epsilon_{t+1}] = f(M_t, z_t) \leq 1, \]
A Life Cycle Model of Household Consumption
- Household Composition

Euler Equation Estimation
- Intuition for Inconsistency
- Constructive Contribution: Bounds

Alternative: Structural (M-)Estimator
- Examples

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Concluding Discussion
Euler Equation Estimation of $\theta$

- Existing literature: Ignore credit constraints, assuming $\lambda_s = 0 \forall s$
- A non-linear GMM approach could be to estimate $\theta$ from the empirical moment

$$\frac{1}{NT} \sum_i^N \sum_t^T \left( R \beta \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{-\rho} \exp(\theta \Delta z_{i,t}) - 1 \right) \cdot Z_{i,t} = 0,$$

where $Z$ is instrument(s)
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where $Z$ is instrument(s)

- Most existing studies work with a log-linearized Euler equation

$$\Delta \log C_{it} = \text{constant} + \rho^{-1} \theta' \Delta z_{it} + \tilde{\epsilon}_{it},$$
Euler Equation Estimation of $\theta$

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$$
\Delta \log C_{it} = \text{constant} + \rho^{-1} \theta' \Delta z_{it} + \tilde{\epsilon}_{it},
$$

- I show that when consumers are potentially credit constrained, estimation of $\theta$ from either method is severely biased. (not always upwards...)

Consumption and Children, T. Jørgensen.
Illustration: Simulation Setup

- Structural parameters are calibrated to “standard” values

<table>
<thead>
<tr>
<th></th>
<th>$R$</th>
<th>$\sigma^2_\varepsilon$</th>
<th>$\sigma^2_\eta$</th>
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<th>$\varphi$</th>
<th>$\mu$</th>
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<td>.8</td>
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- Arrival of children is estimated using the PSID figures
- Households initiated with zero wealth at age 22 and no children
Age Profiles and Intuition for Bounds

(a) Income and Consumption

(b) Euler Residual, ε

Using older households and changes in number of children, ∆z_{it}, produces a lower bound. Using young and cohort-average level of number of children, ∆z_{it}, as an instrument produces an upper bound.
Age Profiles and Intuition for Bounds

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Using older households and changes in number of children, $\Delta z_{it}$, produces a lower bound.
Age Profiles and Intuition for Bounds

(e) Income and Consumption

(f) Euler Residual, $\epsilon$

1. Using *older* households and changes in number of children, $\Delta z_{it}$, produces a **lower bound**

2. Using *young* and *cohort-average* level of number of children, $\Delta \overline{z}_{it}$, as an instrument produces an **upper bound**
Graphical Illustration of Bounds

- **Lower Bound** ($\Delta z_{it}$): including households from 59 through 25

\[\begin{align*}
\Delta z_{it} & \\
\Delta z_{i} & \\
\theta_{0} & \\
\end{align*}\]
Graphical Illustration of Bounds

- **Upper Bound** ($\Delta \overline{z}_{it}$): including households from 25 through 59
- **Lower Bound** ($\Delta z_{it}$): including households from 59 through 25
A Life Cycle Model of Household Consumption
  - Household Composition

Euler Equation Estimation
  - Intuition for Inconsistency
  - Constructive Contribution: Bounds

Alternative: Structural (M-)Estimator
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Concluding Discussion
Structural Estimation (NFXP M-estimator)

- For a given set of structural parameters, \( \Theta = (\beta, \rho, \gamma, \theta) \), the model is solved numerically yielding \( C^*_t(M_t, P_t, z_t|\Theta) \).
Structural Estimation (NFXP M-estimator)

- For a given set of structural parameters, $\Theta = (\beta, \rho, \gamma, \theta)$, the model is **solved numerically** yielding $C^*_t(M_t, P_t, z_t|\Theta)$.
- Define a function of the **observed data**, $O = (M, P, C, z)^{data}$, and **model solution** as
  $$\xi_{it}(\Theta) \equiv \xi(O_{it}, C^*_t|\Theta)$$
  in which the model-predictions for a given observation, $O_{it}$, can be interpolated from the model solution.
Structural Estimation (NFXP M-estimator)

- For a given set of structural parameters, $\Theta = (\beta, \rho, \gamma, \theta)$, the model is solved numerically yielding $C_t^*(M_t, P_t, z_t|\Theta)$.
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  in which the model-predictions for a given observation, $O_{it}$, can be interpolated from the model solution.
- Define another function taking all time observations as argument
  \[ g_i(\Theta, \phi) \equiv g(\xi_i(\Theta), \phi), \]
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- For a given set of structural parameters, \( \Theta = (\beta, \rho, \gamma, \theta) \), the model is solved numerically yielding \( C_t^*(M_t, P_t, z_t|\Theta) \).
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- Define another function taking all time observations as argument
  \[
  g_i(\Theta, \phi) \equiv g(\xi_i(\Theta), \phi),
  \]
  such that,
  \[
  (\hat{\Theta}, \hat{\phi}) = \arg \min_{(\Theta, \phi) \in \mathcal{C}} N^{-1} \sum_{i=1}^{N} g_i(\Theta, \phi)
  \]
  produce consistent and asymptotically Normally distributed parameters under fairly standard regularity conditions.
Example: Additive Normal Measurement Error

If consumption data is contaminated with \textit{iid} additive $\mathcal{N}(0, \sigma_{\xi}^2)$ measurement error, then

\[
\xi_{it}(\Theta) = C_{it}^{data} - \tilde{C}^*(O_{it}|\Theta),
\]

\[
g_i(\Theta, \sigma_{\xi}) = T_i \log(2\pi \sigma_{\xi}^2) + \frac{1}{2\sigma_{\xi}^2} \sum_t \xi_{it}(\Theta)^2,
\]

\[
(\hat{\Theta}, \hat{\sigma}_{\xi}) = \arg\min_{\Theta, \sigma_{\xi}} \frac{1}{N} \sum_i \left\{ T_i \log(2\pi \sigma_{\xi}^2) + \frac{1}{2\sigma_{\xi}^2} \sum_t \xi_{it}(\Theta)^2 \right\},
\]

is the structural parameters that maximize the likelihood of observed data being generated from the structural model.
Example: Multiplicative Log-Normal Heterogeneous Measurement Error

If consumption data is contaminated with multiplicative $\log N(\nu_i, \sigma_\xi^2)$ measurement error systematically different across households, then

$$
\xi_{it}(\Theta) = \log C_{it}^{\text{data}} - \log \tilde{C}^*(O_{it}|\Theta),
$$

$$
g_i(\Theta, \sigma_\xi) = T_i \log(4\pi \sigma_\xi^2) + \frac{1}{4\sigma_\xi^2} \sum_t (\Delta \xi_{it}(\Theta))^2,
$$

$$
(\hat{\Theta}, \hat{\sigma}_\xi) = \arg\min_{\Theta, \sigma_\xi} \frac{1}{N} \sum_i \left\{ T_i \log(4\pi \sigma_\xi^2) + \frac{1}{4\sigma_\xi^2} (\Delta \xi_{it}(\Theta))^2 \right\},
$$

produce consistent estimates of $\Theta$ and $\sigma_\xi^2$. 

Alternative:

$$
g_i(\Theta) = \sum_{t|\Delta \xi_{it}(\Theta)} \text{produce a LAD-type estimator, robust to outliers, impose no distributional assumptions, and allow for heterogeneous measurement error}.
$$
Example: Multiplicative Log-Normal Heterogeneous Measurement Error

If consumption data is contaminated with multiplicative $\log \mathcal{N}(\nu_i, \sigma^2_\xi)$ measurement error systematically different across households, then

$$\xi_{it}(\Theta) = \log C_{it}^{data} - \log \tilde{C}^*(O_{it}|\Theta),$$

$$g_i(\Theta, \sigma_\xi) = T_i \log(4\pi\sigma^2_\xi) + \frac{1}{4\sigma^2_\xi} \sum_t (\Delta \xi_{it}(\Theta))^2,$$

$$\left(\hat{\Theta}, \hat{\sigma}_\xi \right) = \arg\min_{\Theta, \sigma_\xi} \frac{1}{N} \sum_i \left\{ T_i \log(4\pi\sigma^2_\xi) + \frac{1}{4\sigma^2_\xi} (\Delta \xi_{it}(\Theta))^2 \right\},$$

produce consistent estimates of $\Theta$ and $\sigma^2_\xi$.

- Alternative: $g_i(\Theta) = \sum_t |\Delta \xi_{it}(\Theta)|$ produce a LAD-type estimator, robust to outliers, impose no distributional assumptions, and allow for heterogeneous measurement error.
Example: Matching Moments

- Define $p$ moments or auxiliary parameters (ap) from the data, $\lambda$, and the corresponding moments from the predictions of the model, $\lambda(\Theta)$, and let

$$\xi(\Theta) = \lambda - \lambda(\Theta),$$
$$g_i(\Theta, \phi) = \xi(\Theta)'W(\phi)^{-1}\xi(\Theta),$$

where $W(\phi)$ is some weight matrix, produce an estimable equation within the framework of equation (2).

- Note, no simulation of synthetic data is performed here, either.
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Very Short on Data

**Danish Data:**
- Entire population from 1987-1996.
- Stable couples in which wife is aged 25-59 years old.
- Consumption is imputed as
  \[ C_t = Y_t - \Delta A_t, \]
  where disposable income, \( Y \), is *all* earnings and net-wealth, \( A \), is all assets minus all liabilities, except pension funds.

**PSID:**
- After tax household income
- Assets (excluding housing) and no records on liabilities
Estimation specification

- Assume additive normal measurement error in \textit{normalized} consumption,

\[
\xi_{it}(\Theta) = \frac{(C_{it}^{\text{data}} - \tilde{C}^* (O_{it} | \Theta))}{P_{it}^{\text{data}}},
\]

\[
g_i(\Theta, \sigma_\xi) = T_i \log(2\pi\sigma_\xi^2) + \frac{1}{2\sigma_\xi^2} \sum_t \xi_{it}(\Theta)^2,
\]

\[
(\hat{\Theta}, \hat{\sigma}_\xi) = \arg\min_{\Theta, \sigma_\xi} \frac{1}{N} \sum_i^N \left\{ T_i \log(2\pi\sigma_\xi^2) + \frac{1}{2\sigma_\xi^2} \sum_t \xi_{it}(\Theta)^2 \right\},
\]

- Results are robust to changing this specification
Model Fit: additive normal measurement error

(g) Mean consumption

(h) Median consumption

(i) Low skilled

(j) High skilled

Consumption and Children, T. Jørgensen.
## Structural Estimation Results: add. normal errors

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td></td>
<td>Low skilled</td>
<td>High skilled</td>
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<td>High skilled</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(1)</td>
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<tr>
<td>( \rho ) Risk aversion</td>
<td>2.316</td>
<td>2.363</td>
<td>2.385</td>
<td>2.639</td>
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<tr>
<td></td>
<td>(0.041)</td>
<td>(0.036)</td>
<td>(0.043)</td>
<td>(0.057)</td>
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<tr>
<td>( \beta ) Discount factor</td>
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<td>0.964</td>
<td>0.973</td>
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<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
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<tr>
<td>( \gamma ) Retirement</td>
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<td>1.492</td>
<td>1.491</td>
<td>1.254</td>
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<td></td>
<td>(0.018)</td>
<td>(0.018)</td>
<td>(0.020)</td>
<td>(0.022)</td>
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<tr>
<td>( \sigma_{\xi} ) Meas. err.</td>
<td>0.468</td>
<td>0.468</td>
<td>0.468</td>
<td>0.490</td>
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<tr>
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<td>(0.000)</td>
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<td>(0.011)</td>
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$\vartheta(z; \theta) = \exp(\theta^T z)$

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<tr>
<th>$\theta$</th>
<th># of children</th>
<th>-0.017</th>
<th>0.004</th>
<th>0.224</th>
<th>0.326</th>
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<tr>
<td></td>
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<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.098)</td>
<td>(0.361)</td>
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<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
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\( v(z; \theta) = \exp(\theta'z) \)

\( \theta \) # of children

| \( \theta_{11} \) 1. child \( \leq 10 \) | -0.017 | 0.004 | 0.224 | 0.326 |
|                | (0.012) | (0.003) | (0.098) | (0.361) |

\( v(z; \theta) = 1 + \theta'z \)

\( \theta_{12} \) 1. child \( > 10 \)

| \( \theta_{21} \) 2. child \( \leq 10 \) | -0.004 | -0.008 | 0.117 | -0.087 |
|                | (0.013) | (0.010) | (0.288) | (0.549) |

\( \theta_{22} \) 2. child \( > 10 \)

| \( \theta_{31} \) 3. child \( \leq 10 \) | -0.031 | 0.002 | 0.353 | 0.247 |
|                | (0.004) | (0.008) | (0.292) | (0.530) |

| \( \theta_{32} \) 3. child \( > 10 \) | -0.034 | -0.015 | 0.158 | 0.158 |
|                | (0.005) | (0.008) | (0.221) | (0.521) |

| \( -\mathcal{L}(\theta) \) | 0.46536 | 0.46533 | 0.46529 | 0.49868 | 0.49863 | 0.49862 | 0.83299 | 0.83269 | 0.83263 |
| LR [p-val] | 67.1[0.00] | 135.4[0.00] | 57.1[0.00] | 68.8[0.00] | 8.7[0.00] | 10.4[0.11] | 1.3714 | 1.13714 | 1.13710 |
| Observations | 851249 | 430703 | 8333 | 8672 |
Log-Linear Euler Equation Estimation: Bounds

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<th>Low Skilled</th>
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<th>High Skilled</th>
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<td>Lower†</td>
<td>Upper‡</td>
<td>Δzt</td>
<td>Δz̅t</td>
<td>Lower†</td>
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<td>(0.003)</td>
<td>(0.009)</td>
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<td>(0.015)</td>
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<td>(0.073)</td>
<td>(0.024)</td>
<td>(0.060)</td>
<td>(0.024)</td>
<td>(0.186)</td>
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</tbody>
</table>

- sample split at age 45
- if $\rho = 2$:
  - US: $\theta \in [0.04, 0.20]$ for low skilled and $\theta \in [0.12, 0.31]$ for high skilled households
  - DK: $\theta \in [0.05, 0.29]$ and $\theta \in [0.04, 0.37]$ for low and high skilled households, respectively
Discussion and Future Work

1. Shown that method applied in *all* existing studies produce inconsistent estimates of the value of children

2. Supply something like bounds
   - easily estimated using the methods used in the literature (OLS/IV) on young/old samples

3. Propose alternative structural M-estimator that is very flexible

4. Apply the estimation approach on Danish and US data
   - finds very small –if any– effects of children
   - compared to the literature where $\theta \approx .8$ (outside of my bounds)

In the Future:

1. The Monte Carlo results suggest significant downwards bias when using full sample.
   $\rightarrow$ but the literature report estimates much larger than MC results.

2. Ideas...?
References I


Simulating Children Profile

(k) Infant Arrival Probabilities, PSID

Figure 1: Estimated Arrival Probability of Infant (PSID) and Simulated Number of Children.
Recursive Bounds, data

(a) Upper Bound, Denmark

(b) Upper Bound, PSID

(c) Lower Bound, Denmark

(d) Lower Bound, PSID

Figure 2: Estimated Upper and Lower Bounds for Varying Cut-off Ages.

Consumption and Children, T. Jørgensen.
Robustness

<table>
<thead>
<tr>
<th></th>
<th>Danish Registers</th>
<th></th>
<th>PSID</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low skilled</td>
<td>High skilled</td>
<td>Low skilled</td>
<td>High skilled</td>
</tr>
<tr>
<td></td>
<td>Logistic</td>
<td>Abs</td>
<td>Logistic</td>
<td>Abs</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.565 (0.076)</td>
<td>0.588 (0.054)</td>
<td>1.272 (0.219)</td>
<td>1.148 (0.000)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.973 (0.000)</td>
<td>0.974 (0.000)</td>
<td>0.976 (0.002)</td>
<td>0.982 (0.000)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.152 (0.013)</td>
<td>1.010 (0.010)</td>
<td>1.300 (1.300)</td>
<td>1.300 (1.300)</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>0.421 (0.000)</td>
<td>0.444 (0.001)</td>
<td>0.483 (0.005)</td>
<td>0.626 (0.007)</td>
</tr>
<tr>
<td>$\theta_{11}$</td>
<td>0.001 (0.004)</td>
<td>0.001 (0.007)</td>
<td>0.201 (0.091)</td>
<td>0.181 (0.149)</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>-0.008 (0.002)</td>
<td>-0.002 (0.005)</td>
<td>0.269 (0.085)</td>
<td>0.172 (0.121)</td>
</tr>
<tr>
<td>$\theta_{21}$</td>
<td>-0.010 (0.003)</td>
<td>-0.003 (0.005)</td>
<td>0.057 (0.062)</td>
<td>0.044 (0.120)</td>
</tr>
<tr>
<td>$\theta_{22}$</td>
<td>0.006 (0.003)</td>
<td>0.003 (0.005)</td>
<td>0.075 (0.082)</td>
<td>0.099 (0.126)</td>
</tr>
<tr>
<td>$\theta_{31}$</td>
<td>0.006 (0.005)</td>
<td>0.005 (0.008)</td>
<td>0.258 (0.116)</td>
<td>0.234 (0.141)</td>
</tr>
<tr>
<td>$\theta_{32}$</td>
<td>0.014 (0.007)</td>
<td>0.006 (0.011)</td>
<td>0.092 (0.131)</td>
<td>0.098 (0.262)</td>
</tr>
</tbody>
</table>
Extra: Income Growth

Figure 3: Age Profile of Income Growth, $\hat{G}_t$. 
Extra: Calibrations

- **Permanent Income,** $P_t$, is uncovered using the Kalman Filter

<table>
<thead>
<tr>
<th></th>
<th>Denmark</th>
<th>PSID</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_t$</td>
<td>Fig. 3</td>
<td>Own calculations: see text</td>
</tr>
<tr>
<td>$R$</td>
<td>1.03</td>
<td>Gourinchas and Parker (2002)</td>
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<tr>
<td>$\kappa$</td>
<td>−0.6</td>
<td>Own calculations: see text</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.10</td>
<td>Own calculations: see text</td>
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<tr>
<td>$\mu$</td>
<td>0.30</td>
<td>Own calculations: see text</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.90</td>
<td>Ministry of Finance (2003)</td>
</tr>
<tr>
<td>$G_{ret}$</td>
<td>1.00</td>
<td>Own calculations: see text</td>
</tr>
</tbody>
</table>

Age-profiles

Consumption and Children, T. Jørgensen.
**Extra: Calibrations, income variances**

Table 2: Permanent and Transitory Income Shock Variances.

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Low skilled</th>
<th>High skilled</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est</td>
<td>SE</td>
<td>Est</td>
</tr>
<tr>
<td>$\sigma^2_\eta$</td>
<td>0.0054</td>
<td>(0.000096)</td>
<td>0.0049</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.0072</td>
<td>(0.000156)</td>
<td>0.0059</td>
</tr>
</tbody>
</table>

*Danish Registers*

<table>
<thead>
<tr>
<th></th>
<th>All</th>
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<th>High skilled</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est</td>
<td>SE</td>
<td>Est</td>
</tr>
<tr>
<td>$\sigma^2_\eta$</td>
<td>0.0785</td>
<td>(0.003898)</td>
<td>0.0756</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.0510</td>
<td>(0.004452)</td>
<td>0.0476</td>
</tr>
</tbody>
</table>

*PSID*

**Notes:** Estimates based on the approach in Meghir and Pistaferri (2004).
Arrival of Infant

High skilled

Low skilled

0 children
1 child
2 children

Estimated
Model Math

\[ u(C_t) = v(z_t; \theta)(1 - \rho)^{-1}C_t^{1-\rho}, \]
\[ V_{T+1}(M_{T+1}) = \gamma v(z_{T+1}; \theta)(1 - \rho)^{-1}(M_{T+1} + hP_{T+1})^{1-\rho}, \]
\[ M_{t+1} = R(M_t - C_t) + Y_{t+1}, \]
\[ M_t = A_t + C_t, \]
\[ A_t \geq \kappa P_t, \ \forall t, \]
\[ Y_t = P_t \varepsilon_t, \ \log \varepsilon_t \sim N(-\sigma^2/2, \sigma^2), \]
\[ P_t = G_t P_{t-1} \eta_t, \ \log \eta_t \sim N(-\sigma^2/2, \sigma^2). \]