Numerical Integration

Numerical example to illustrate Matlab, mex and OpenMP implementation: Numerical Integration, using "discrete sum".

$$\int_0^1 f(x)dx \approx \frac{1}{N}\sum_{i=1}^{N} f(x_i),$$

where

$$x_i = \frac{i - 0.5}{n}.$$

Here, let

$$f(x) = \frac{4}{1 - x^2},$$

since we know that

$$\int_0^1 \frac{4}{1 - x^2} dx = \pi.$$

Numerical Precision

The precision on a computer is finite! Hence, a lot of information (digits) are cut off when performing operations.
All numbers on a computer is represented as

$$number = significant \times base^{exponent},$$

where *base* can be 2, 10 or 16. (From WIKI, I have no idea...)
Using $base = 10$, the number 12.05 can be represented as

$$12.05 = .1205 \times 10^2$$

Table: Double Precision (Floating Point)

bites:	1	11	52	= 64
use:	sign	exponent	significant	

Numerical Precision

When performing operations, the two numbers has to be normalized to have the same exponent, reducing precision!

For example say we only have 7 significant digits (double precision gives approx. 16) and want to add the two numbers,

$$\begin{aligned}
354.05 + 210,000.1 &= .35405 \times 10^3 + .2100001 \times 10^6 \\
&= .00035405 \times 10^6 + .2100001 \times 10^6 \\
&= (.0003540 + .2100001) \times 10^6 \\
&= .2103541 \times 10^6 \\
&= 210,354.1 \neq 210,354.15
\end{aligned}$$

where *chopping* 5 gives a result different than the true sum! Hence, ordering of sum matters (non-associative):

$$(a + b) - c \neq (a - c) + b$$