A Model for Retirement
Theory and Estimation

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- Methode for analyzing retirement used in latest report from Danish Economics Councils (Link: http://www.dors.dk/graphics/Synkron-Library/Publikationer/Rapporter/For%E5r_2013/Trykt/F13_Kapitel_3.pdf)

- Collaboration between Danish Economics Councils (DØRS) and DREAM (Working paper: http://www.dors.dk/graphics/Synkron-Library/Publikationer/Arbejdspapirer/Arbejdspapir%20om%20tilbagetrakningsmodel.pdf)

Stock & Wise (1990)

Income for a person retiring at age r:

$$Y_s \text{ for } s < r$$

$$B_s(r) \;\; for s \geq r$$

Stock & Wise (1990)

Utility function:

$$U_w(Y_s) = Y_s^{\gamma} + w_s$$

$$U_r(B_s(r)) = (k\, B_s(r))^{\gamma} + \xi_s$$

where

$$w_s = \rho\, w_{s-1} + \epsilon_{ws}$$

$$\xi_s = \rho\, \xi_{s-1} + \epsilon_{\xi s}$$

Stock & Wise (1990)

Remarks:

- Utility is a function of income not consumption (= credit rationing or myopic behavior). No consumption smoothing
- Utility function is not general (Y^γ is used in stead of $\frac{1}{\gamma}Y^\gamma$)
- Utility function is stocastic (health costs)
- Parameter k is identical for all

Arnberg & Stephensen (2013)

- Perfect capital market. Consumption smoothing
- Generel utility function (CRRA)
- Utility function not stocastic (only time of death is random)
- Parameter k is not identical for all (random coefficient)

Income

As S&W: Income for a person retiring at age r:

$$Y_s \text{ for } s < r$$

$$B_s(r) \quad for s \geq r$$

Individual $B_s(r)$ are calculated using all the tricks of the trade.

The Consumer: Inspired by S&W

Utility at age 0, given retirement at age r :

$$U_0(r;k) = \phi V_0(r;k) + \epsilon_r$$

$$V_0(r;k) = \sum_{s=1}^{T} \frac{(\gamma_s(r;k)\, c_s)^{1-\rho}}{1-\rho} \beta_s$$

$$\gamma_s(r;k) = \begin{cases} 1 & \text{for } s < r \\ k > 1 & \text{for } s \geq r \end{cases}$$

$$\beta_t \equiv \prod_{s=0}^{t} \frac{1-\mu_s}{1+\theta}$$

The Consumer (cont...)

Budget restriction:

$$A_s = (1 + i_s)A_{s-1} + y_s - c_s$$

The interest rate is given by (similar to Yaari (1965) and Blanchard (1985)):

$$(1 + i_s)(1 - \mu_s) \equiv 1 + i$$

Income:

$$y_s = y_s(r) = \begin{cases} Y_s & \textit{for } s < r \\ B_s(r) & \textit{for } s \geq r \end{cases}$$

Analytical solution

Optimal bahavior:

$$U_0\left(r;k\right)=\phi\frac{\hat{V}_0\left(r;k\right)^{1-\rho}}{1-\rho}+\epsilon_r$$

Indirect utility function:

$$\hat{V}_0\left(r;k\right)=\frac{A_0+H_0\left(r\right)}{P_0\left(r;k\right)}$$

$$H_0\left(r\right)\equiv\sum_{s=1}^{T}y_s\left(r\right)R_s,\ \ R_s=\prod_{v=0}^{s}\frac{1-\mu_v}{1+i}$$

$$P_0\left(r;k\right)\equiv\left[\sum_{s=1}^{T}\gamma_s\left(r;k\right)^{\frac{1-\rho}{\rho}}\beta_s^{\frac{1}{\rho}}R_s^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

Deterministic case

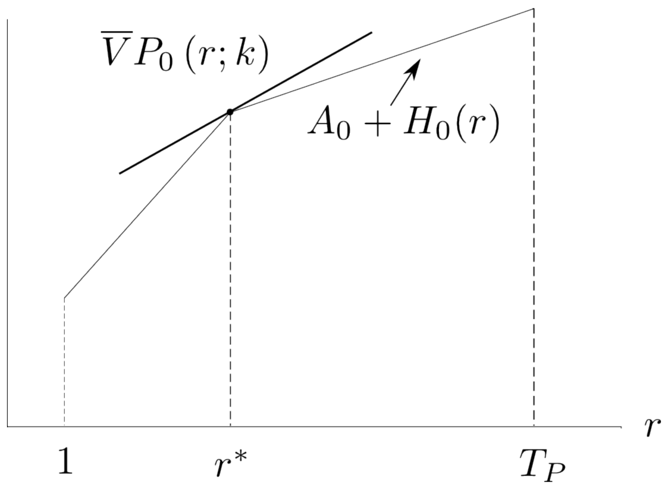
Indifference curve:

$$\hat{V}_0(r;k) = \frac{A_0 + H_0(r)}{P_0(r;k)} = \overline{V}$$

such that:

$$A_0 + H_0(r) = \overline{V} P_0(r;k)$$

Deterministic case: Indifference curves



Estimation: Statistical Model

- Utility based discrete-choise-model with random coefficient:

$$U_0\left(r;k\right) = \phi V_0\left(r;k\right) + \epsilon_r$$

  Could choose many models: logit, probit, mixed logit osv.

- We choose the simplest: logit model with random coefficient (k) => the ϵ_r'-s are independent extreme value distributed

Logit assumption

The logit assumption implies that:

$$P\left(r|k,\phi\right)=\frac{exp\left(\phi\frac{\hat{V}_0(r;k)^{1-\rho}}{1-\rho}\right)}{\sum_{s=1}^{T_P}exp\left(\phi\frac{\hat{V}_0(s;k)^{1-\rho}}{1-\rho}\right)}$$

Estimation of random coefficient

- Non-parametric estimation of variation in k.
- Method similar to Kenneth Train (2007) “A Recursive Estimator for Random Coeffcient Models”
- The method exploites that:

$$\text{Expected conditional distribution} = \text{distribution}$$

Estimation of random coefficient

Assume a population distribution $p(k)$. This is the distribution we are looking for.
Define for person j :

$$P_j(r_j|k_j) = \pi(r_j|k_j;\phi, x_j)$$

where π is the logit-specification and

$$x_j = \left(A_0^j + H_0^j(1), ..., A_0^j + H_0^j(T_P)\right)$$

- We know x_j, r_j
- We do not know k_j

Expected Conditional Distribution = Distribution

Conditional distribution: how is k_j distributed given what person j actually did? Bayes rule:

$$P_j\left(k_j|r_j\right) = \frac{\pi\left(r_j|k_j;\phi,x_j\right)p\left(k\right)}{P\left(r_j;\phi,x_j\right)}$$

where

$$P\left(r_j;\phi,x_j\right) \equiv \int_1^\infty \pi\left(r_j|\kappa;\phi,x_j\right)p\left(\kappa\right)d\kappa$$

or

$$P_j\left(k_j|r_j\right) \propto \pi\left(r_j|k_j;\phi,x_j\right)p\left(k\right)$$

Expected Conditional Distribution = Distribution

You can show that (I tried - and Train did):

$$E\left[P_j\left(k|r_j\right)\right] = p\left(k\right)$$

$$p\left(k\right) \Rightarrow P_j\left(k|r_j\right) \Rightarrow E\left[P_j\left(k|r_j\right)\right] \simeq \frac{1}{N}\sum_{j=1}^{N} P_j\left(k|r_j\right) \Rightarrow p\left(k\right)$$

This implies that $p\left(k\right)$ is a fix point.
Strong intuition: if you take the average of everybodys best guess on the k-distribution, you get the k-distribution.

Fix Point: Successive over-relaxation

How do we find the fix point?:

- Start with some distribution $p^1(k)$
- In iteration i : Calculate for all j : $P_j^i(k|r_j) = \frac{\pi\left(r_j|k;\phi,x_j\right)p^i(k)}{P\left(r_j;\phi,x_j\right)}$
- In iteration i : Calculate new distribution $p^{i+1}(k) = \lambda p^i(k) + (1-\lambda)\sum_{j=1}^{n} P_j^i(k|r_j)$
- Continue till convergence

Analysis: Data

Danish register data for the cohorte that had age 59 in 2001

For each individual j we know

- time of retirement r_j
- Income Y_{js} for $s < r_j$
- Pensions $B_s(r_j)$ for $s \geq r_j$

We caluculate

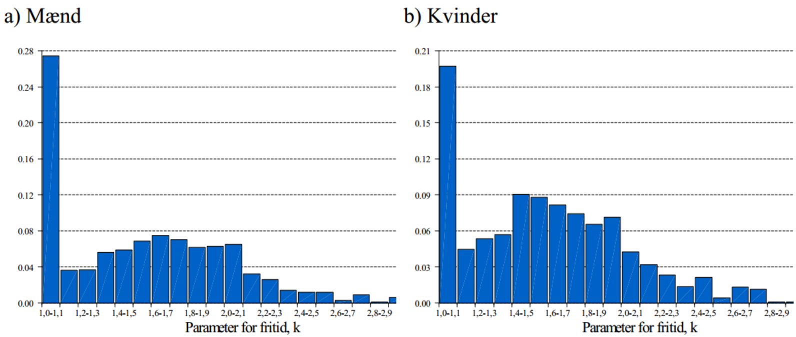
- Income Y_{js} for $s \geq r_j$ (using avarage age-profile)
- Pensions $B_s(r)$ for $r \neq r_j$ (using actualrial math)

Analysis: The k-distribution

	Estimate
1. quartile	1.35
Median	1.57
3.quartile	1.82

Remark: We assume $\rho = 2$. It is estimated that $\phi = 8$ mio.

Analysis: The k-distribution



Analysis: Retirement (pct.)

Age	Actual	Expected
60	31	30
61	4	3
62	26	19
63	8	6
64	4	4
65	10	7
66	4	12
67+	13	20

Further research

- Peer-group-effect
- Age-effect (k increases as you get older)